Ahmad, J., K. Buchardt, & C. Furrer (2022). Computation of bonus in multi-state life insurance. *ASTIN Bulletin* **52**(1), pp. 291-331. doi: 10.1017/asb.2021.32.

Corrigendum

1 October, 2023

Corrections

- Replace

$$\mathrm{d}B^{\circ,\mathrm{f}}(t) = \sum_{j\in\mathcal{J}^{\mathrm{f}}} \mathbb{1}_{(Z(t-)=j)}\left(b_{j'}(t)^{+}\,\mathrm{d}t + \sum_{\substack{k\in\mathcal{J}^{\mathrm{f}}\\ k\neq j}} b_{j'k'}(t)^{+}\,\mathrm{d}N_{jk}(t)\right)$$

  on p. 297 by

$$\begin{aligned}\mathrm{d}B^{\circ,\mathrm{f}}(t) &= b_{(J+1)(2J+1)}(t)\,\mathrm{d}N_{(J+1)(2J+1)}(t)\\ &\quad + \sum_{j\in\mathcal{J}^{\mathrm{f}}} \mathbb{1}_{(Z(t-)=j)}\left(b_{j'}(t)^{+}\,\mathrm{d}t + \sum_{\substack{k\in\mathcal{J}^{\mathrm{f}}\setminus\{2J+1\}\\ k\neq j}} b_{j'k'}(t)^{+}\,\mathrm{d}N_{jk}(t)\right)\end{aligned}$$

  and carry out the necessary consequential corrections for $A^{\circ}(t,s)$ in (2.8) and (2.9) on p. 299.

- Replace the next to last paragraph of Subsection 2.2 on p. 297 beginning with "It is useful..." and ending with "$B^{\circ,\mathrm{p},\pm}(0) = 0$." by the following: "For later purposes, we also define a benefit payment stream $B^{\circ,+}$ mimicking $B^{\circ,\mathrm{f}}$, but extended to all of $\mathcal{J}$:

$$\begin{aligned}B^{\circ,+}(t) &= B^{\circ,\mathrm{p},+}(t) + B^{\circ,\mathrm{f}}(t),\\ \mathrm{d}B^{\circ,\mathrm{p},+}(t) &= b_{(J+1)(2J+1)}(t)\,\mathrm{d}N_{0J}(t)\\ &\quad + \sum_{j\in\mathcal{J}^{\mathrm{p}}} \mathbb{1}_{(Z(t-)=j)}\left(b_j(t)^{+}\,\mathrm{d}t + \sum_{\substack{k\in\mathcal{J}^{\mathrm{p}}\setminus\{J\}\\ k\neq j}} b_{jk}(t)^{+}\,\mathrm{d}N_{jk}(t)\right),\quad B^{\circ,\mathrm{p},+}(0) = 0.\text{"}.\end{aligned}$$

- Extend

$$b_{0J} = \widetilde{V}_0^{\star},$$

  on p. 298 to

$$\begin{aligned}b_{0J} &= \widetilde{V}_0^{\star},\\ b_{(J+1)(2J+1)} &= \widetilde{V}_0^{\star,+},.\end{aligned}$$

These corrections do not impact the main theoretical results of the paper. Furthermore, the numerical example actually utilizes the correct surrender payments and technical reserves, although this is not apparent from the presentation itself.

Background

In the paper, the technical reserves were supposed to be unaffected by policyholder behavior, that is free policy conversion and surrender. To this end, on p. 298 the following specifications were made:

$$\begin{aligned}
&\mu^{\star}_{jk} = \mu^{\star}_{j'k'}, \quad j,k \in \mathcal{J}^{\mathrm{f}}, k \neq j,\\
&b_{0J} = \widetilde{V}^{\star}_{0},\\
&(0,\infty) \ni t \mapsto \rho(t) = \frac{\widetilde{V}^{\star}_{0}(t)}{\widetilde{V}^{\star,+}_{0}(t)}.
\end{aligned}$$

The superscript + refers to the payment stream $B^{\circ,+}$ which on p. 297 is defined as

$$B^{\circ,+}(t) = B^{\circ,\mathrm{p},+}(t) + \rho(\tau)B^{\circ,\mathrm{f}}(t),$$

where the second term was given by

$$\mathrm{d}B^{\circ,\mathrm{f}} = \sum_{j\in\mathcal{J}^{\mathrm{f}}} \mathbb{1}_{(Z(t-)=j)}\left(b_{j'}(t)^{+}\,\mathrm{d}t + \sum_{\substack{k\in\mathcal{J}^{\mathrm{f}}\\k\neq j}} b_{j'k'}(t)^{+}\,\mathrm{d}N_{jk}(t)\right), \quad B^{\circ,\mathrm{f}}(0) = 0,$$

with $j' = j - (J+1)$, while the first term was given by

$$B^{\circ,\mathrm{p},+}(t) = \sum_{j\in\mathcal{J}^{\mathrm{p}}} \mathbb{1}_{(Z(t-)=j)}\left(b_{j}(t)^{+}\,\mathrm{d}t + \sum_{\substack{k\in\mathcal{J}^{\mathrm{p}}\\k\neq j}} b_{jk}(t)^{+}\,\mathrm{d}N_{jk}(t)\right), \quad B^{\circ,\mathrm{p},+}(0) = 0.$$

In both cases, the ambiguous notation $b_{jk}(t)^{+}$ was taken to mean $\max\{0, b_{jk}(t)\}$. This implied that the surrender payments in $B^{\circ,\mathrm{p},+}$ and $B^{\circ,\mathrm{f}}$ were taken to be the maximum of zero and the technical reserve for the payment stream consisting of both premiums and benefits. We are sorry to report that this specification is insufficient to ensure that the technical reserves are unaffected by free policy conversion and surrender. Instead, these surrender payments should equal the technical reserve for a payment stream consisting solely of benefits that are not reduced by a free policy factor. Furthermore, the definition of $\widetilde{V}^{\star,+}_{j}$ on p. 298 should, similarly, not involve the free policy factor. The changes listed above amend these errors.