UNIVERSITY OF COPENHAGEN
DEPARTMENT OF MATHEMATICAL SCIENCES

# Scandinavian Actuarial Conference 2024


Bringing experts and leaders in actuarial science together from across the world to present the newest and most impactful developments in the field.




**Plenary speakers:**
Valérie Chavez-Demoulin, Damir Filipović, Joshua Loftus, and Johanna Ziegel.

Featuring a roundtable on fairness moderated by the renowned Karel Van Hulle, who is joined by panelists Matthias Fahrenwaldt, Isabelle Flückiger, Esko Kivisaari, and Thomas Møller.

Call for abstracts is open – send your submission before 31 March 2024.

Register before 20 May 2024 to benefit from early bird rates.

Read more, submit, and register on sac24.net

SAC24 is officially starting 15 August 2024. A satellite event to commemorate Ragnar Norberg (1945 – 2017) takes place 14 August from 14:00 till 20:00. "Remembering Ragnar" will be hosted by Mogens Steffensen with talks by Walther Neuhaus, Thomas Møller, Stéphane Loisel, Marcus C. Christiansen, and Kristian Buchardt.

The document template originates from LaTeXTemplates.com and is based on the original version at: https://github.com/maximelucas/AMCOS_booklet.

# Contents

# About

**Welcome to Copenhagen!**

The goal of the Scandinavian Actuarial Conference 2024 is to bring experts and leaders in actuarial science together from across the world, aiming to present the newest and most impactful developments in the field. The conference officially starts on the 15th of August, though a satellite event to commemorate Ragnar Norberg takes place on the 14th of August.

The conference features plenary talks by Valérie Chavez-Demoulin (University of Lausanne), Joshua Loftus (London School of Economics), Johanna Ziegel (ETH Zurich), and Damir Filipović (EPFL and Swiss Finance Institute).

## Useful information

**Talks** are held at the H.C. Ørsted Institute (HCØ) in Auditoriums 1, 5, and 6. The address is Universitetsparken 5, DK-2100 Copenhagen Ø.

The registration and service desk is located outside Aud 1. Next to it, you will find a book exhibition by *Springer*. *Springer* is also offering a 20 % discount for conference attendees on all their books.

**Coffee breaks and lunches** take place just outside Aud 1 and in *Vandrehallen*.

Wi-Fi is available via eduroam or the network *KU Guest*.

The **social event and conference dinner** takes place on the evening of the 15th of August at *Arbejdermuseet* (the Workers Museum). The address is Rømersgade 22, DK-1362 Copenhagen.

## Committees, partners, and sponsors

**Organizing committee:** Martin Bladt, Christian Furrer (chairperson), Natasha Rørdam Gulddal, Munir Eberhardt Hiabu, Mogens Steffensen.

**Scientific committee:** Hansjörg Albrecher, Luis Alvarez Esteban, Fred Espen Benth, Martin Bladt, Marcus C. Christiansen, Boualem Djehiche, Christian Furrer, Munir Eberhardt Hiabu, Filip Lindskog, Thomas Mikosch (chairperson), Jostein Paulsen.

**Partners and sponsors:** Fynske Købstæders Fond, the Carlsberg Foundation (CF23-0914), and the Danish Society of Actuaries.

## Roundtable on fairness

*Insurance addresses inequalities and, consequently, fairness has always been a critical theoretical and practical concern. The insurance industry is expected to deliver solutions based on societal demand and scientific and technological support. Can it keep up – and how? And what is the role of the actuarial scientist and the actuary compared to, say, the emerging data scientists? The topic becomes very relevant in the present debate about social sustainability and the right to be forgotten.*

In the afternoon of Thursday, 15th of August, the conference features a roundtable on fairness moderated by the renowned Karel Van Hulle. He is joined by four panelists from practice: Matthias Fahrenwaldt, Isabelle Flückiger, Esko Kivisaari, and Thomas Møller. The panel will discuss the matter at hand and engage in questions from the audience.

## Social event and conference dinner

After Thursday's roundtable, we leave behind the H.C. Ørsted Institute and proceed to visit *Arbejdermuseet* (the Workers Museum, Rømersgade 22, DK-1362 Copenhagen) and to enjoy a taste of modern Danish cuisine. The following rough outline serves as an indication of what to expect:

| | | | |
|---|---|---|---|
| 17:10 | – | 17:50 | Joint walk from the H.C. Ørsted Institute to *Arbejdermuseet* |
| 17:50 | – | 18:15 | Arrival and welcome drinks |
| 18:15 | – | 19:15 | Introduction to the museum and opportunity to explore the exhibitions |
| 19:15 | – | 21:30 | Conference dinner |
| 21:30 | – | 24:00 | Music, drinks, and *hygge*. |

# Code of Conduct

The event seeks to provide an experience for all participants that is free from harassment, bullying, discrimination, and inappropriate behavior which includes, but is not limited to:

- Intimidation, personal attacks, stalking, following, harassment, inappropriate physical contact or attention, or derogatory mention

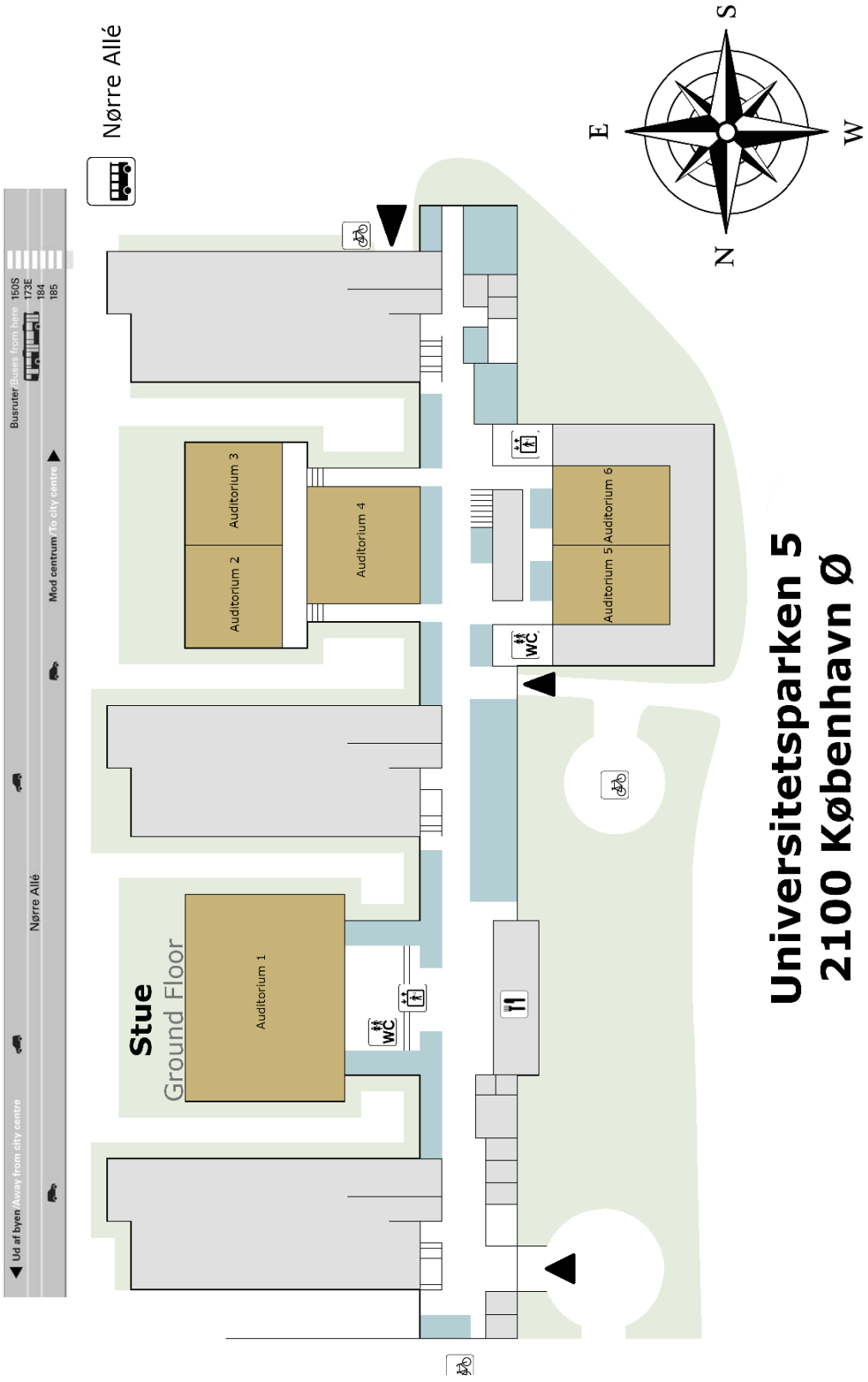
- Offensive comments, materials or activities related to age, race, color, nationality, gender, sexual orientation, medical condition, disability, or pregnancy

- Disorderly, boisterous, or disruptive conduct including fighting, coercion, theft, damage to property, or any mistreatment or unprofessional behavior towards participants.

Participants asked to stop any such behavior are expected to comply immediately, whether at conference venues or conference-related social events. If you are subject to or an observer of inappropriate behavior, please contact the organizer or organizing staff immediately.

In case of a breach to this Code of Conduct, the organizer will respond swiftly towards the offender as deemed appropriate according to law and regulations. Sanctions may be, but are not limited to, a warning, expulsion from the event with no refund, barring from participation in future events, reporting the incident to the offender's organization/company, or in the last instance reporting the incident to law enforcement authorities.

# Venue map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Ground Floor
Auditorium 1
Auditorium 2
Auditorium 3
Auditorium 4
Auditorium 5
Auditorium 6
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Universitetsparken 5
2100 København Ø

# Remembering Ragnar

> "Certain forms of uncertainty make life interesting but less safe. I was always intrigued by the various forms of risk that are associated with human life and activity and how they can be mitigated for the individual by contractual risk exchange between two or more parties. Certain forms of certainty make life interesting and more safe. I was always attracted to mathematics because it allows for statements that are non-trivial and still indisputably true. These two areas of interest synthesize perfectly into actuarial/financial mathematics, which gives precise contents to notions of risk and develops methods for measuring and controlling it."
>
> (Ragnar Norberg, 1945 – 2017)

*Remembering Ragnar* is a half-day event where we commemorate Ragnar Norberg and his many contributions, not least to the actuarial research literature and to the research and educational environments he cultivated, championed, and challenged. The event is hosted by Mogens Steffensen and features talks by Walther Neuhaus, Thomas Møller, Stéphane Loisel, Kristian Buchardt, and Marcus C. Christiansen. It is concluded by a reception dinner.

## Wednesday, 14th of August

| | | | | |
|---|---|---|---|---|
| 13:30–14:00 | **Registration and coffee** | | | |
| 14:00–14:10 | Aud 1 | **Opening** | | |
| 14:10–14:50 | Aud 1 | **Walther Neuhaus** | Remembering Ragnar – the early days | IS |
| 14:50–15:30 | Aud 1 | **Thomas Møller** | Aspects of interplay between finance and insurance | IS |
| 15:30–15:55 | **Coffee and sweets** | | | |
| 15:55–16:35 | Aud 1 | **Stéphane Loisel** | Ragnar in Lyon: retired but not tired! | IS |
| 16:35–17:15 | Aud 1 | **Kristian Buchardt** | Forward transition rates for valuation | IS |
| 17:15–17:55 | Aud 1 | **Marcus C. Christiansen** | Two hidden gems in the works of Ragnar Norberg | IS |
| 17:55–18:00 | Aud 1 | **Closing** | | |
| 18:00–20:00 | **Reception dinner in *Vandrehallen*** | | | |

Abbreviations: CT(s) for Contributed Talk(s); IS for Invited Speaker; PS for Plenary Speaker.

# Timetables

## Thursday, 15th of August

| | | | | |
|---|---|---|---|---|
| 8:00–8:45 | **Registration and coffee** | | | |
| 8:45–9:00 | Aud 1 | **Opening** | | |
| 9:00–10:00 | Aud 1 | **Valérie Chavez-Demoulin** | Causal Discovery in Multivariate Extremes with a Hydrological Analysis of Swiss River Discharges | PS |
| 10:00–10:20 | **Coffee** | | | |
| 10:20–12:00 | Aud 1<br>Aud 5<br>Aud 6 | Machine learning<br>Mortality and longevity risk<br>Valuation and design | | CTs |
| 12:00–13:00 | **Lunch** | | | |
| 13:00–14:00 | Aud 1 | **Joshua Loftus** | Model-agnostic explanation tools and their limitations | PS |
| 14:05–15:20 | Aud 1<br>Aud 5<br>Aud 6 | Fairness<br>Multi-state modeling<br>Dividends and stochastic control | | CTs |
| 15:20–15:45 | **Coffee and sweets** | | | |
| 15:45–17:00 | Aud 1 | **Roundtable on fairness** | | |
| 18:00–24:00 | **Social event and conference dinner at *Arbejdermuseet*** | | | |

## Friday, 16th of August

| | | | | |
|---|---|---|---|---|
| 9:00–10:00 | Aud 1 | **Johanna Ziegel** | Isotonic distributional regression and CRPS decompositions | PS |
| 10:00–10:20 | **Coffee** | | | |
| 10:20–12:00 | Aud 1<br>Aud 5<br>Aud 6 | Optimal consumption, investment, and insurance<br>Economics of (re)insurance<br>Statistics and extremes | | CTs |
| 12:00–13:00 | **Lunch** | | | |
| 13:00–14:40 | Aud 1<br>Aud 5<br>Aud 6 | Non-life insurance<br>Asset allocation<br>Lapse risk | | CTs |
| 14:40–15:00 | **Coffee and sweets** | | | |
| 15:00–16:00 | Aud 1 | **Damir Filipović** | Stripping the Swiss Discount Curve using Kernel Ridge Regression | PS |
| 16:00–16:10 | Aud 1 | **Closing** | | |

# Parallel sessions — Thursday, 15th of August

## Machine learning — Aud 1, 10:20–12:00

| | Chair: Munir Eberhardt Hiabu | | |
|---|---|---|---|
| 10:20–10:45 | **Ralf Wilke** | Estimation of Panel Models with Group Structures in Fixed Effects | CT |
| 10:45–11:10 | **Mathias Lindholm** | A tree-based varying coefficient model with insurance applications | CT |
| 11:10–11:35 | **Gabriele Pittarello** | A machine learning approach based on survival analysis for IBNR frequencies in non-life reserving | CT |
| 11:35–12:00 | **Thorsten Rheinländer** | Deep Learning in Life Insurance | CT |

## Mortality and longevity risk — Aud 5, 10:20–12:00

| | Chair: Mogens Bladt | | |
|---|---|---|---|
| 10:20–10:45 | **Cinzia Di Palo** | Entropy and Life Annuity Changes | CT |
| 10:45–11:10 | **Phillip Frede Halmsted Olsen** | Matrix representations for prices of life-contingent derivatives | CT |
| 11:10–11:35 | **Andrey Ugarte Montero** | Incorporating Information on Insured Amounts to Improve Survival Rate Estimates from a Liability Perspective | CT |
| 11:35–12:00 | **Mathias D. Plovst** | Systematic longevity risk: The willingness to pay | CT |

## Valuation and design — Aud 6, 10:20–12:00

| | Chair: Luis Alvarez Esteban | | |
|---|---|---|---|
| 10:20–10:45 | **Francesco Della Corte** | Market-Consistent Valuation and Capital Assessment for Demographic Risk in Life Insurance: A Cohort Approach | CT |
| 10:45–11:10 | **Francesco Rania** | Risk profiles of Reverse Mortgage: empirical evidence from Italy | CT |
| 11:10–11:35 | **Daniel Nkameni** | Index based contracts as a component of an optimal insurance coverage against emerging risks | CT |
| 11:35–12:00 | **Michèle Vanmaele** | Second order Esscher pricing for models with jumps | CT |

## Fairness — Aud 1, 14:05–15:20

| | Chair: Mathias Lindholm | | |
|---|---|---|---|
| 14:05–14:30 | **Giovanni Rabitti** | Measuring proxy discrimination effects with sensitivity analysis | CT |
| 14:30–14:55 | **Tessa Steensgaard** | Fair learning and testing for unfairness given protected features | CT |
| 14:55–15:20 | **Xi Xin** | Why You Should Not Trust Interpretations in Machine Learning: Adversarial Attacks on Partial Dependence Plots | CT |

## Multi-state modeling — Aud 5, 14:05–15:20

| | Chair: Christian Furrer | | |
|---|---|---|---|
| 14:05–14:30 | **Ayşe Arık** | Examining breast cancer risk during COVID-19: Insights from semi-Markov modelling | CT |
| 14:30–14:55 | **Oliver L. Sandqvist** | Likelihood-based estimation for multistate models subject to IBNR- and RBNS effects | CT |
| 14:55–15:20 | **Theis Bathke** | Estimation of bivariate transition rates in life insurance | CT |

## Dividends and stochastic control — Aud 6, 14:05–15:20

| | Chair: Jostein Paulsen | | |
|---|---|---|---|
| 14:05–14:30 | **Dante Mata Lopez** | On an optimal dividend problem with a concave bound on the dividend rate | CT |
| 14:30–14:55 | **Zuoquan Xu** | Optimal dividend payout under path-dependent constraint | CT |
| 14:55–15:20 | **Indira Dhar** | Stochastic Control Problems in the Dynamic Nelson-Siegel Framework | CT |

# Parallel sessions — Friday, 16th of August

## Optimal consumption, investment, and insurance — Aud 1, 10:20–12:00

| | Chair: Mogens Steffensen | | |
|---|---|---|---|
| 10:20–10:45 | **Seyoung Park** | Optimal Consumption and Investment Decisions with Disastrous Income Risk: Revisiting Rietz's Rare Disaster Risk Hypothesis | CT |
| 10:45–11:10 | **Debbie Kusch Falden** | Calibration of risk aversion to real pension asset allocation | CT |
| 11:10–11:35 | **Julie Bjørner Søe** | Optimal consumption, investment, and life insurance, including state dependence by risk-adjusted utility | CT |
| 11:35–12:00 | **Philipp C. Hornung** | Investigating trade-offs in the design of smooth pension products | CT |

## Economics of (re)insurance — Aud 5, 10:20–12:00

| | Chair: Filip Lindskog | | |
|---|---|---|---|
| 10:20–10:45 | **Svein-Arne Persson** | Why firms should buy insurance | CT |
| 10:45–11:10 | **Tao Li** | (A)symmetric Information and Insurers' Nitpicky Behavior | CT |
| 11:10–11:35 | **Lea Enzi** | Stochastic differential reinsurance games: a jump filtration approach | CT |
| 11:35–12:00 | **Aleksandar Arandjelovic** | Reinsurance with neural networks | CT |

## Statistics and extremes — Aud 6, 10:20–12:00

| | Chair: Thomas Mikosch | | |
|---|---|---|---|
| 10:20–10:45 | **Jorge Yslas** | Robust claim frequency modeling through phasetype mixture-of-experts regression | CT |
| 10:45–11:10 | **Yubo Rasmussen** | Analyzing Extreme Weather Impact on Property Insurance Claim Severity: A Combined Pareto Neural Network Model | CT |
| 11:10–11:35 | **Christoffer Øhlenschlæger** | Heterogeneous extremes in the presence of random covariates and censoring | CT |
| 11:35–12:00 | **Jiajun Liu** | An Asymptotic study of the generalized Tail-Gini measures | CT |

## Non-life insurance — Aud 1, 13:00–14:40

| | Chair: Martin Bladt | | |
|---|---|---|---|
| 13:00–13:25 | **Uwe Schmock** | On Matrix-Valued Gamma Distributions in Multivariate Poisson Mixture Models | CT |
| 13:25–13:50 | **Melanie Averhoff** | Experience Rating in the Cramér-Lundberg Model | CT |
| 13:50–14:15 | **Taariq Nazar** | On duration effects in non-life insurance pricing | CT |
| 14:15–14:40 | **Filip Lindskog** | Mack's estimator motivated by large exposure asymptotics in a compound Poisson setting | CT |

## Asset allocation — Aud 5, 13:00–14:40

| | Chair: Thorsten Rheinländer | | |
|---|---|---|---|
| 13:00–13:25 | **Yukio Muromachi** | A Term Structure Model of Interest Rates with Regime-Switching Properties for Risk Evaluation | CT |
| 13:25–13:50 | **Yevhen Havrylenko** | Asset-liability management with liquid and fixed-term assets | CT |
| 13:50–14:15 | **Carlos Miguel Glória** | Optimal strategy for an AAM of DC pension plans under jump-diffusion and with time-varying ambiguity | CT |
| 14:15–14:40 | **Michael Preisel** | Long-Term Mean-Variance Optimization Under Mean-Reverting Equity Returns | CT |

## Lapse risk — Aud 6, 13:00–14:40

| | Chair: Marcus C. Christiansen | | |
|---|---|---|---|
| 13:00–13:25 | **Andrea Molent** | Enhancing Valuation of Variable Annuities in Lévy Models with Stochastic Interest Rate | CT |
| 13:25–13:50 | **Oytun Haçarız** | Lapse-supported Life Insurance and Adverse Selection | CT |
| 13:50–14:15 | **Francesco Ungolo** | A Dirichlet Process Mixture regression model for the analysis of competing risk events | CT |
| 14:15–14:40 | **Stefan Gerhold** | The effect of policy cancellation on the risk of an insurance portfolio | CT |

# List of abstracts

## Reinsurance with neural networks

***Aleksandar Arandjelović***[1], ***Julia Eisenberg***[2]

CT

[1] Institute for Statistics and Mathematics, WU Wien, Austria
[2] Institute for Statistics and Mathematical Methods in Economics, TU Wien, Austria

We consider an insurance company who faces financial risk in form of insurance claims and random market-dependent income fluctuations. The company aims to simultaneously control it's terminal wealth (e.g. at the end of an accounting period) and minimal available capital during this period by purchasing reinsurance. The multidimensional target functional consists of a utility function penalised by an extended version of a Gerber–Shiu function. The optimal reinsurance policy, maximizing the target functional, is parametrized via neural networks. The procedure is illustrated by a numerical example, where the surplus process is given by a Cramér–Lundberg model perturbed by a mean-reverting Ornstein–Uhlenbeck process.

## Examining breast cancer risk during COVID-19: Insights from semi-Markov modelling

***Ayşe Arık***[1], ***Andrew Cairns***[1], ***Erengul Dodd***[2], ***Angus S Macdonald***[1], ***George Streftaris***[1]

CT

[1] Department of Actuarial Mathematics and Statistics, Heriot-Watt University, and the Maxwell Institute for Mathematical Sciences, United Kingdom
[2] School of Mathematical Sciences, University of Southampton, United Kingdom

Public health measures imposed due to the COVID-19 pandemic have seriously affected cancer pathways. This impact includes, for instance, halting screening or delaying diagnostic tests. The focus of this study is on breast cancer, which is one of the most common cancer morbidity and mortality among women. Specifically, we are interested in quantifying the impact of transitioning from observed and treated pathways to unobserved and untreated pathways on breast cancer mortality as a result of COVID-related health measures. We introduce a semi-Markov model for women aged 65-89 years in England. Our model incorporates events related to cancer diagnosis and progression based on publicly available population data and relevant medical literature. We quantify age-specific excess deaths, for a period up to 5 years, along with years of life expectancy lost and changes in cancer survival by cancer stage. Our analysis suggests a 3–6% increase in breast cancer deaths, and a 4–6% increase in registrations of advanced breast cancer, findings that remain robust under sensitivity analysis.

## Experience Rating in the Cramér-Lundberg Model

***Melanie Averhoff*** [1], ***Julie Thøgersen*** [1,2]

CT

[1] Department of Economics and Business Economics, Aarhus University, Denmark
[2] Department of Mathematics, Aarhus University, Denmark

This paper provides a study of how experience rating on both claim frequency and severity impacts the solvency of an insurance business in the continuous-time Cramér Lundberg model. This is done by treating the claim parameters as random outcomes and continuously updating the premiums using Bayesian estimators. In the analysis, the claim sizes conditional on the severity parameter are assumed to be light-tailed. The main contributions are large deviation results, where the asymptotic ruin probabilities are found for a model updating the premium based upon both frequency and severity. This asymptotic ruin probability is compared to the one of a model, which updates the premium solely based on claim frequency. Our findings are illustrated with a parametric example, where the conditional claim size are exponentially distributed, and the severity parameter is the outcome of gamma distribution.

## Estimation of bivariate transition rates in life insurance

***Theis Bathke***

CT

Institute of Mathematics, Carl von Ossietzky Universität Oldenburg, Germany

In survival analysis, the estimation of bivariate survival functions was introduced to model dependencies in event time for different but related individuals, see [2]. Dependencies between different jump times of one individual also arise in non-Markov multi-state modelling. To model these dependent jumps, we use bivariate transition rates and transition probabilities. These can be appropriately estimated in a non-Markov multi-state model with censored data using conditional Nelson-Aalen and Aalen-Johansen estimators. These bivariate transition rates and transition probabilities can then be used to estimate reserves for path-dependent life insurance cash flows as they occur at policy changes or to estimate second moments of one-dimensional cash flows with deterministic payment functions, see [1].

[1] Bathke, T., Christiansen, M.C. Two-dimensional forward and backward transition rates. Eur. Actuar. J. (2023). https://doi.org/10.1007/s13385-023-00363-3.

[2] Gill, R. D. "Multivariate survival analysis." Theory of Probability & Its Applications 37.2 (1993): 284-301.

## Forward transition rates for valuation

***Kristian Buchardt*** IS

AP Pension, Copenhagen, Denmark
Department of Mathematical Sciences, University of Copenhagen, Denmark

Ragnar Norberg popularised his predecessor in Copenhagen Jan M. Hoem's multi-state approach to life and pension insurance valuation. Today, this approach is widely used in practice. One key area of ongoing development is allowing for stochastic transition rates and the associated quest towards a definition of forward transition rates, the hope being to adopt the elegance of the forward interest rate. Ragnar Norberg made a significant impact in this area. In this talk, we consider – in the context of modern life and pension insurance in practice – the history of forward transition rates, Ragnar Norberg's contribution, and later developments.

## Causal Discovery in Multivariate Extremes with a Hydrological Analysis of Swiss River Discharges

***Linda Mhalla*** [2], ***Valérie Chavez-Demoulin*** [1], ***Philippe Naveau*** [3] PS

[1] HEC, University of Lausanne, Switzerland
[2] Institute of Mathematics, EPFL, Switzerland
[3] CNRS LSCE, France

Understanding the causal dynamics between hydrological factors at their extreme level is crucial for effective water resources management, flood risk assessment and climate change adaptation efforts. Causal asymmetry is based on the principle that an event is a cause only if its absence would not have been a cause. From there, uncovering causal effects becomes a matter of comparing a well-defined score in both directions. Motivated by studying causal effects at extreme levels of a multivariate random vector, we propose to construct a model-agnostic causal score relying solely on the assumption of the existence of a max-domain of attraction. Based on a representation of a Generalized Pareto random vector, we construct the causal score as the Wasserstein distance between the margins and a well-specified random variable. The proposed methodology is illustrated on a hydrologically simulated dataset of different characteristics of catchments in Switzerland: discharge, precipitation, and snowmelt [1].

[1] Mhalla, L. and Chavez-Demoulin, V. and Naveau, P. (2024). Causal Discovery in Multivariate Extremes with a Hydrological Analysis of Swiss River Discharges. *arXiv:2405.10371*.

## Two hidden gems in the works of Ragnar Norberg

***Marcus C. Christiansen*** IS

Institute of Mathematics, Carl von Ossietzky Universität Oldenburg, Germany

We look at two pioneering contributions by Ragnar Norberg, the potential of which has long been overlooked by the actuarial community. The first result is the 'stochastic Thiele equation' published in 1992, and the second contribution is a definition of 'forward transition rates' published in 2010. Both contributions go beyond the limited world of classical Markov modelling and consider fundamental concepts of life mathematics from the more general perspective of martingale theory. Recent research builds on this pioneering work with the aim of unifying the fragmented modelling landscape of Markov models, semi-Markov models, higher order Markov models, etc. into a common intuitive framework.

[1] Norberg, R. (1992). Hattendorff's theorem and Thiele's differential equation generalized. *Scandinavian Actuarial Journal* **1**, pp. 2–14.

[2] Norberg, R. (2012). Forward mortality and other vital rates – Are they the way forward? *Insurance: Mathematics and Economics* **47**, pp. 105–112.

## Market-Consistent Valuation and Capital Assessment for Demographic Risk in Life Insurance: A Cohort Approach

***Gian Paolo Clemente*** [1], ***Francesco Della Corte*** [1], ***Nino Savelli*** [1], ***Diego Zappa*** [2] CT

[1] Department of Mathematics for Economic, Financial and Actuarial Sciences, Catholic University of the Sacred Heart, Milan, Italy
[2] Department of Statistical Sciences, Catholic University of the Sacred Heart, Milan, Italy

We explore the quantification of demographic risk in accordance with the market-consistent valuation set by Solvency II. Our contribution includes closed formulas for assessing the inflows and outflows of an insurance policy portfolio utilizing a cohort approach (see [1]). To maintain versatility, we address both traditional and equity-linked policies, offering a market-consistent valuation of liabilities (see [2]). Subsequently, we compute the Solvency Capital Requirement for idiosyncratic risk (linked to accidental mortality) and systematic risk (trend risk), presenting a formal closed formula for the former and an algorithm for the latter. Our analysis reveals that the accidental volatility of policyholders' deaths is influenced by the inherent characteristics of the cohort's policies (Sums-at-Risk), the age of policyholders, and the variability of sums insured. Furthermore, trend risk is contingent on both accidental volatility and the longevity forecasting model employed.

[1] Clemente, G.P., Della Corte, F., Savelli, N. (2022). A stochastic model for capital requirement assessment for mortality and longevity risk, focusing on idiosyncratic and trend components. *Annals of Actuarial Science* **16**, 527–546.

[2] Wüthrich, M. V., Bühlmann, H., and Furrer, H. and others (2010). Market-consistent actuarial valuation. *Springer* **2**.

## Stochastic Control Problems in the Dynamic Nelson-Siegel Framework

***Indira Dhar*** [1], ***Ralf Korn*** [1,2]

CT

[1] Department of Mathematics, Rheinland-Pfälzische Technische Universität Kaiserslautern-Landau, Germany
[2] Department of Financial Mathematics, Fraunhofer ITWM, Germany

In this paper, we consider a stochastic model of the yield curve in the dynamic Nelson-Siegel framework. We then analyze the stochastic control problems as done in [1], in the framework of this dynamic Nelson-Siegel model. Further, we analyze the impact of different yield curves on portfolio problems under this setting.

[1] Korn, R., and Kraft, H. (2002). A stochastic control approach to portfolio problems with stochastic interest rates. *SIAM Journal on Control and Optimization* **40(4)**, pp. 1250–1269.

## Stochastic differential reinsurance games: a jump filtration approach

***Lea Enzi***, ***Stefan Thonhauser***

CT

Institute of Statistics, Graz University of Technology, Austria

Stochastic differential games in insurance business have already been studied in various ways. One example is given by the competition of two insurance companies, which are referred to as players, on a common insurance market. Each of the players wealth is represented by a surplus process which can individually be controlled by acquiring reinsurance. The two companies competition is based on a reward function, with one player aiming to maximize it and the other trying to minimize it. While most of the papers make use of a diffusion approximation to make the problem more tractable, we take an alternative approach by exploiting the special structure of the original underlying processes. We consider the stochastic differential game in the form “strategy against control", introduced in the stochastic setting by Fleming and Souganidis [1]. We show that the upper and lower value functions are viscosity solutions of corresponding Bellman-Isaacs equations. There, both the form of the processes and, more specifically, the underlying jump filtration are crucial. Finally, we illustrate the interaction between the two players in a numerical example.

[1] Fleming, W. H., Souganidis, P. E. (1989). On The Existence of Value Functions of Two-Player, Zero-Sum Stochastic Differential Games. *Indiana University Mathematics Journal* **38**(2), pp. 293–314.

## Calibration of risk aversion to real pension asset allocation

***Jens-Philip Dehn-Toftehøj*** [2], ***Debbie Kusch Falden*** [1,3], ***Mogens Steffensen*** [3]

CT

[1] Department of Mathematical Sciences, University of Liverpool, United Kingdom
[2] Datasolvr ApS, Copenhagen, Denmark
[3] Department of Mathematical Sciences, University of Copenhagen, Denmark

An investor's risk aversion is a fundamental element in financial decision-making and preferences but lacks a standardised calibration method. We propose an approach to measure the risk aversion of an investor managing a diverse portfolio that includes pension savings, real estate, and free funds. We utilise the investor's real asset allocation as the optimal strategy, assuming the investor's preferences follow a power utility function (CRRA utility function). We calibrate a risk aversion parameter by building on Merton's formulas for optimal investment strategies [3, 4]. For the pension savings, we account for the present value of future premiums, which results in an optimal investment strategy consistent with real life-cycle pension products as described in [5], [1] and [2]. Realistic and stable risk aversions are calibrated by constructing a customised risky fund aligned with the investor's preferences. Disparities in risk aversion across financial categories are examined by certainty equivalents, and a numerical study with a real Danish pension portfolio emphasises the practical applications of our results.

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## Stripping the Swiss Discount Curve using Kernel Ridge Regression

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PS

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We analyze and implement the kernel ridge regression (KR) method developed in Filipovic, Pelger and Ye (2022) to estimate the risk-free discount curve for the Swiss government bond market. We show that the insurance industry standard Smith–Wilson method is a special case of the KR framework. We recapitulate the curve estimation methods of the Swiss Solvency Test (SST) and the Swiss National Bank (SNB). In an extensive empirical study covering the years 2010 to 2022 we compare the KR curves with the SST and SNB curves. The KR method proves to be robust, flexible, transparent, reproducible and easy to implement, and outperforms the benchmarks in- and out-of-sample. We show the limitations of all methods for extrapolating the yield curve and propose possible solutions for the extrapolation problem. We conclude that the KR method is the preferred method for estimating the discount curve. Link: `https://ssrn.com/abstract=4611310`.

## The effect of policy cancellation on the risk of an insurance portfolio

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CT

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We investigate a quantitative justification of the lapse fee, i.e. the difference between cash value and surrender value of an insurance contract, by studying the increased risk of the average loss of an insurance portfolio when a policy is cancelled. The question is motivated by recent Austrian jurisdiction and an ongoing discussion in the German actuarial community. An asymptotic analysis of value-at-risk is connected to the classical Cornish-Fisher expansion, and leads to a new generalization for expected shortfall, and other coherent measures of risk. Our main results apply to losses from the domain of attraction of the normal distribution. We also discuss the problem for non-Gaussian stable distributions, where the results look different and depend on the index of stability.

## Optimal strategy for an AAM of DC pension plans under jump-diffusion and with time-varying ambiguity

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CT

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Business Research Unit, Portugal

This paper investigates the robust optimal investment for an ambiguity averse member (AAM) of a defined contribution (DC) pension plan in a fully-fledged, time consistent mean-variance modeling framework. In particular, the paper extends the literature in defined contribution (DC) pension

plans in 3 directions: 1) We relax its assumption of purely continuous stock and/or contribution processes, which allows to introduce the effects of news, job loss, macroeconomic conditions, etc., into the model; 2) Unlike most studies in DC pension plans, we allow for ambiguity about both the mean arrival rate and jump size distribution of the contribution rate process of the member; and 3) Ambiguity in our setting is time-varying. The model thus features stochastic volatility, stochastic interest rate, stochastic contribution rate, jumps in both stock and contribution rate processes, and time-varying ambiguity. The framework proposed in this paper is general and adds significant realism to existing models in the literature.

## Lapse-supported Life Insurance and Adverse Selection

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CT

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Using expected lapses in pricing life insurance is a way of obtaining a competitive edge by reducing insurance premiums. But if lapses are less than expected, insurers will face losses, which might be increased by a form of adverse selection. If individuals at the highest mortality risk are also least likely to lapse a life insurance policy, then lapse-supported premiums magnify adverse selection costs. We give an expression for the cost of the reduction in premiums supported by lapses, which we believe is new, and consider the consequences of adverse selection, using a 'Term to 100' contract for numerical illustrations. We assume a heterogeneous population in which: (a) insurers cannot identify individuals at high mortality risk; (b) a secondary market exists that prevents high-risk policies from lapsing; (c) financial underwriting is lax or absent; and (d) life insurance policies may even be initiated by third parties as a financial investment ('stranger-originated life insurance' or STOLI). Adverse selection losses under (a) are typically very small, but under (b) can be increased by multiples, and under (c) and (d) increased almost without limit. We note that the different approaches to modeling lapses used in studies of adverse selection and genetic testing appear to be broadly equivalent and robust.

## Asset-liability management with liquid and fixed-term assets

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CT

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Insurance companies and pension funds have asset-allocation processes that may involve multiple risk management constraints due to liabilities. Furthermore, the investment universe of such institutional investors often contains assets with different levels of liquidity, e.g., liquid stocks and illiquid investments in infrastructure projects or private equity. Therefore, we propose an analytically tractable framework for economic agents who maximize their expected utilities by choosing investment-consumption strategies subject to lower bound constraints on both inter-

mediate consumption and the terminal value of assets, some of which are liquid, while others are fixed-term. Our framework extends [1] to consumption and risk management constraints. Combining the generalized martingale approach with the ideas from [2] and [3], we derive optimal decisions in our framework and analyze them from an insurance economics perspective.

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## Investigating trade-offs in the design of smooth pension products

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CT

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We investigate different ways to design smooth pension products based on solutions from optimal consumption and investment problems. Smoothness of a consumption process can be studied from both a pathwise (measured in terms of quadratic variation) and a pointwise (measured in terms of variance) point of view and we conclude that introducing one type of smoothing does not necessarily improve the other type of smoothing. Thus care has to be taken when designing smooth pension products. Focusing on pathwise smoothness without disregarding pointwise smoothness, we provide both a qualitative as well as a quantitative discussion of the trade-offs involved. In the qualitative discussion we find that to increase smoothness it is necessary either reduce the starting value, the drift of consumption or the level of terminal wealth.

For the quantitative discussion we set up an optimal consumption and investment problem, where the first control is the proportion of wealth invested into the risky asset, but the second control is not the consumption process itself. Instead we use the drift and volatility of consumption as controls. The objective is to minimise the quadratic distance to a target drift and volatility, while introducing a penalty on the volatility. We find explicit solutions to this problem using classic dynamic programming methods and use them to study the three trade-offs theoretically and numerically. All three approaches result in both pointwise and pathwise smoothing compared to the target, but reducing the drift yields better pointwise smoothing for similar levels of pathwise smoothing.

## (A)symmetric Information and Insurers' Nitpicky Behavior

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[2] School of Finance, Renmin University of China, China
[3] Department of Economics and Finance, University of Guelph, Canada

CT

This paper delves into the widespread perception of insurers as bad payers, often accused of unjustly rejecting legitimate claims. We explore the economic mechanisms leading to this negative image by examining the strategic "nitpicking" behavior of insurers. Such behavior involves an insurer's action to cut the payments made on honest claims. Our findings reveal that this nitpicking behavior only arises in markets with asymmetric information, where policyholders are unable to observe insurers' nitpicking strategies. Conversely, in markets with symmetric information, insurers lose the incentive to engage in nitpicking. Moreover, our study highlights that nitpicking activities can reduce social welfare in a monopolistic competition market and even lead to market failure. This is because nitpicking is essentially an overpriced security that charges lower premiums from policyholders at no loss state, but reduces actual indemnities received by policyholders at loss state. Further, we also expand our model to include insurance fraud. Our findings advocate for regulatory interventions, suggesting that publicizing complaints against insurers could expose and mitigate their nitpicky incentives, thereby enhancing social welfare and market efficiency.

## A tree-based varying coefficient model with insurance applications

***Mathias Lindholm***, ***Henning Zakrisson***

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CT

We discuss the tree-based varying coefficient model (VCM) from [3] where the varying coefficients are modelled using the cyclic gradient boosting machine (CGBM) from [1]. Modelling the coefficient functions using a CGBM allows for dimension-wise early stopping and feature importance scores. The dimension-wise early stopping not only re- duces the risk of dimension-specific overfitting, but also reveals differences in model complexity across dimensions. The use of feature importance scores allows for simple feature selection and easy model interpretation. The model is evaluated on both simulated and real insurance data examples, producing out of sample losses comparable to those of the neural network-based VCM called LocalGLMnet from [2].

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## Mack's estimator motivated by large exposure asymptotics in a compound Poisson setting

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The distribution-free chain ladder of Mack justified the use of the chain ladder predictor and enabled Mack to derive an estimator of conditional mean squared error of prediction for the chain ladder predictor. Classical insurance loss models, i.e. of compound Poisson type, are not consistent with Mack's distribution-free chain ladder. However, for a sequence of compound Poisson loss models indexed by exposure (e.g. number of contracts), we show that the chain ladder predictor and Mack's estimator of conditional mean squared error of prediction can be derived by considering large exposure asymptotics. Hence, quantifying chain ladder prediction uncertainty can be done with Mack's estimator without relying on the validity of the model assumptions of the distribution-free chain ladder.

[1] Engler, N., Lindskog, F. (2024). Mack's estimator motivated by large exposure asymptotics in a compound Poisson setting. *Astin Bulletin*, forthcoming.

## An Asymptotic study of the generalized Tail-Gini measures

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[2] School of Statistics, Xi'an University of Finance and Economics, China

In this talk, we explore the generalized Tail-Gini measures by considering the risk attitude. Specifically, we focus on the extreme case with $q$ approaching $1$ and investigate the asymptotic behavior of these Tail-Gini risk measures in scenarios with heavy-tailed and asymptotically dependent/independent risks. We derive explicit asymptotic formulas that clearly illustrate how the tail dependence structure and margins impact the limit behavior of the proposed tail risk measures. Additionally, we provide numerical studies to demonstrate and substantiate our theoretical findings.

## Model-agnostic explanation tools and their limitations

***Joshua Loftus*** [1], ***Lucius Bynum*** [2], ***Sakina Hansen*** [1], ***Kateryna Koval*** [3], ***Anna-Alina Bondarets*** [3] PS

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[2] Center for Data Science, New York University, United States
[3] Faculty of Applied Sciences, Ukrainian Catholic University, Ukraine

Tools for interpretable machine learning or explainable artificial intelligence can be used to audit algorithms for fairness or other desired properties. In a "black-box" setting–one without access to the algorithm's internal structure–the methods available to an auditor may be model-agnostic.

These methods are based on varying inputs while observing differences in outputs, and include some of the most popular interpretability tools like Shapley values and Partial Dependence Plots. Such explanation methods have important limitations. Moreover, their limitations can impact audits with consequences for outcomes such as fairness.

Our work [1] highlights key lessons that regulators, auditors, or other users of model-agnostic explanation tools must keep in mind when interpreting their output. Although we focus on a selection of tools for interpretation and on fairness as an example auditing goal, the lessons we highlight generalize to many other applications of model-agnostic explanations. These tools are increasing in popularity, which makes understanding their limitations an important research direction. That popularity is driven largely by their ease of use and portability. In high-stakes settings like an audit, however, it may be worth the extra work to use tools that can incorporate background information and be tailored to each specific application. We highlight some promising ways to integrate background information by using causal modeling [2, 4, 3].

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[2] Loftus, J., Bynum, L., and Hansen, S. (2024). Causal Dependence Plots. *Forthcoming*.

[3] Kusner, M., Loftus, J., Russell, C., and Silva, R. (2017). Counterfactual fairness. *Advances in neural information processing systems*.

[4] Loftus, J. (2023). It's About Time: Counterfactual Fairness and Temporal Depth. *European Workshop on Algorithmic Fairness*.

## Ragnar in Lyon: retired but not tired!

***Stéphane Loisel*** IS

LIRSA, CNAM, France

In this talk we explain the role that Ragnar had at Institut de Science Financière et d'Assurances at University Lyon 1. We then present the projects in which he was involved and some additional results that were obtained following discussions with Ragnar. We conclude with a few personal 'anecdotes'.

## On an optimal dividend problem with a concave bound on the dividend rate

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CT

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[2] Département de finance, École des sciences de la gestion, Université de Québec à Montréal (UQAM), Canada

We study a version of De Finetti's optimal dividend problem where the surplus of a company is driven by a general diffusion until the first passage time below level zero. In our version, the control strategies are assumed to have an absolutely continuous density with respect to the Lebesgue measure, which is bounded above by an increasing, concave function of the surplus. We provide sufficient conditions to prove that an optimal strategy exists and lies within the set of generalized refraction strategies. We are able to characterise the candidate optimal refraction barrier in our setting, and prove rigorously that indeed it is an optimal strategy. Consequently, we provide an analytical expression for the associated value function. We illustrate the theoretical results with numerical computations and examples.

## Enhancing Valuation of Variable Annuities in Lévy Models with Stochastic Interest Rate

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This paper extends the valuation and optimal surrender framework for variable annuities with guaranteed minimum benefits in a Lévy equity market environment by incorporating a stochastic interest rate described by the Hull-White model. This approach frames a more dynamic and realistic financial setting. We introduce a robust valuation mechanism employing a hybrid numerical method that merges tree methods for interest rate modeling with finite difference techniques for the underlying asset price. This method is particularly effective for addressing the complexities of variable annuities, where periodic fees and mortality risks are significant factors. Our findings reveal the influence of stochastic interest rates on the strategic decision-making process concerning the surrender of these financial instruments. Through comprehensive numerical experiments, and by comparing our results with those obtained by using the Longstaff-Schwartz Monte Carlo method, we illustrate how our refined model can guide insurers in designing contracts that equitably balance the interests of both parties. This is particularly relevant in discouraging premature surrenders while adapting to the realistic fluctuations of financial markets. Lastly, a comparative statics analysis with varying interest rate parameters underscores the impact of interest rates on the cost of the optimal surrender strategy, emphasizing the importance of accurately modeling stochastic interest rates.

## Incorporating Information on Insured Amounts to Improve Survival Rate Estimates from a Liability Perspective

***Andrey Ugarte Montero*, *Torsten Kleinow*, *Frank van Berkum*, *Michel Vellekoop*** CT

Research Center for Longevity Risk, University of Amsterdam, Netherlands

Insurance companies need statistical tools to adequately assess the value of the risk associated with their liabilities. In the life insurance industry in particular, survival modelling is key to accurately assessing the value of insurance policies and annuity business. Traditional techniques, however, emphasize individual survival over time, regardless of the impact that an individual may have on liabilities based on their sum insured. As a result, practitioners have resorted to different methods to account for the fact that discrepancies between actual and expected survival of individuals with higher sum insured may be more critical to a company's liabilities than those of individuals with lower benefits. In this context, our research focuses on formalizing and analyzing in depth some of the ways that can be used in the insurance industry to account for the role of the sum insured in developing survival models. As part of the study we use a new private dataset with survival information of individuals buying annuity products in the Netherlands to investigate how weighing observations with the sum assured or pension benefit will impact mortality estimates and financial predictions. In our analysis, we focus on both well-established techniques based on maximum likelihood estimation with classical mortality laws and generalized linear (additive) models, which allow to account for the risk factors available in our dataset when modelling mortality.

## A Term Structure Model of Interest Rates with Regime-Switching Properties for Risk Evaluation

***Yukio Muromachi*** CT

Department of Management, Tokyo Metropolitan University, Japan

In order to evaluate the risk of a financial institutions such as a bank quantitatively, a simultaneous evaluation of its various kinds of assets and liabilities should be desirable, and the interest rate model used for the purpose should be able to express both the risk under a usual economic state and the risk under a stressed economic state with appropriate probabilities. And, the analyses of the long-term historical data of interest rates and credit spreads would imply the existence of several different economic regimes, specifically, a calm regime with lower default risk and volatility, and a stressed regime with higher default risk and volatility.

Based on such observations, we propose a joint risk evaluation model of interest rate risk and credit risk with the Markovian regime-switching property. We discuss (1) the dynamics of the regime, interest rate and default intensity under a physical probability measure (or, a real probability measure), and (2) the change of measure to a convenient pricing measure such as the risk-neutral probability measure, and (3) the term structures of interest rates with different credit ratings on the future economic scenarios. For simplicity, we assume the regime switches independently of other stochastic processes. In (2), the market prices of interest rate risk, credit risk and regime-switching risk are all included in the model parameters. From (1) and (2), the dynamics of stochastic processes under the pricing measure can be derived, and in (3), through the no-arbitrage pricing of the

default-free (defaultable) bonds, the term structures of the default-free (defaultable) interest rates are obtained at present and in future.

We propose a tractable model and its preliminary results. Although the model looks too simple in a mathematical sense (some parameters are constants and/or linear), the stochastic movement of the regime in the future generates various shapes of the future yield curves with different credit ratings, and the yield curves are highly correlated through the regime. We also demonstrate the risk evaluation of some assets and liabilitiess by using the Monte Carlo simulation. We hope that such models will reveal some new perspectives for financial risk management, especially for asset liability management (ALM). Additionally, if the economic regimes estimated from data include highly stressed one, we can set our model parameters consistent with the data, then, the model becomes consistent with the stress tests. In such a sence, our model might become a bridge between the mathematical/statistical risk evaluation models and the stress tests.

Unfortunately, the calibration of our model parameters is not so easy because various kinds of data are necessary and some useful time-series data cannot be observed yet at present. In order to avoid the difficulty, we introduce a few practical simple assumptions, and propose a stepwise calibration method which is easy to execute. Since our present calibration results are not so bad, but not so good, more sophisticated calibration methods are desired.

## Aspects of interplay between finance and insurance

***Thomas Møller*** IS

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In the middle of the 1990s Ragnar Norberg started to study financial mathematics and its applications in insurance. At this time, the concept of market values in life insurance was not yet very well understood, and early papers treated the insurance risks and the financial risks somewhat separately. During the following years, the literature on applications of financial mathematics in insurance was booming, and Norberg later referred to these developments as the “financial mathematics bubble in insurance”. In this talk we review some early results on hedging and valuation from this period, where insurance risks and financial risks are analyzed by using quadratic hedging approaches. The applications include the hedging of longevity risk via mortality derivatives, in a model framework with both systematic and unsystematic insurance risk. We conclude by showing that the current Danish accounting practice for taxes is in general conservative, when considered in an idealized setting with symmetric and continuously-paid taxes.

## On duration effects in non-life insurance pricing

***Mathias Lindholm, Taariq Nazar***

CT

Department of Mathematics, Stockholm University, Sweden

The paper discusses duration effects on the consistency of mean parameter and dispersion parameter estimators in exponential dispersion families (EDFs) that are the standard models used for non-life insurance pricing. Focus is on the standard generalised linear model assumptions where both the mean and variance, conditional on duration, are linear functions in terms of duration. We derive simple convergence results that highlight consequences when the linear conditional moment assumptions are not satisfied. These results illustrate that: (i) the resulting mean estimators always have a relevant asymptotic interpretation in terms of the duration adjusted actuarially fair premium – a premium that only agrees with the standard actuarial premium using a duration equal to one, given that the expected value is linear in the duration; (ii) deviance based estimators of the dispersion parameter in an EDF should be avoided in favour of Pearson estimators; (iii) unless the linear moment assumptions are satisfied, consistency of dispersion and plug-in variance estimators can not be guaranteed and may result in spurious over-dispersion.

The results provide explicit conditions on the underlying data generating process that will lead to spurious over-dispersion that can be used for model checking. This is illustrated based on real insurance data, where it is concluded that the linear moment assumptions are violated, which results in non-negligible spurious over-dispersion.

## Remembering Ragnar – the early days

***Walther Neuhaus***

IS

ISEG, University of Lisbon, Portugal

I will speak about Ragnar's work in Oslo up to about 1989. He was a teacher at Oslo University. His scientific work of the time focused mainly on credibility theory. Less known is the profound influence that Ragnar had on the Norwegian pre-Solvency II insurance regulation, and how that regulation predated Solvency II in many respects. Ragnar had a formative influence on this speaker since we met at University of Oslo in 1978. I will use the opportunity to speak a little about my own actuarial interests and their roots in Ragnar's work: credibility theory, equalisation schemes and claim cost estimation.

## Index based contracts as a component of an optimal insurance coverage against emerging risks

***Olivier Lopez, Daniel Nkameni*** CT

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Emerging risks are a significant concern in the insurance sector due to their complexity in assessment and coverage using traditional insurance mechanisms [1]. Recent studies suggest that index-based insurance holds promise in mitigating emerging risks [2]. This insurance model operates by using predefined indices that trigger the compensation process and determine the compensation amount without prior knowledge of actual losses. The simplified application of index-based insurance reduces costs and delays associated with indemnity-based insurance expertise. However, despite its benefits, index-based insurance encounters challenges in addressing severe losses, leading to increased basis risk [3]. This issue is particularly pronounced in heavy-tailed losses, common in cyber insurance and insurance against natural disasters. This paper explores the optimal combination of index-based insurance with indemnity-based insurance contracts to leverage the strengths of both approaches. A threshold is introduced to determine when the different types of guarantees activate. Our theoretical and practical analyses contribute to designing this coverage by optimizing the policyholder's utility. We validate our findings using real data from agricultural and cyber insurance contexts. The adopted model could facilitate discussions and create a consensus between insurers and insured around index-based solutions.

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## Matrix representations for prices of life-contingent derivatives

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We consider the pricing of equity linked insurance contracts in a Black-Scholes market, where the residual life of a policyholder follows a phase-type distribution. Notably, the phase-type assumption offers advantages in notation and generality compared to previously considered models, wherein the time until death was assumed to have a distribution where the density is a linear combination of exponentials. At the applied probabilistic level, several novelties emerge in the context of phase-type theory, where we utilize techniques from the theory of Bernstein functions and functions

of matrices. As an example, we derive an explicit representation for a multivariate phase-type distribution describing the joint distribution of the running maximum and its distance from the current level of a Brownian motion stopped at a phase-type distributed time.

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## Entropy and Life Annuity Changes

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Mortality improvements, especially in adults and old age, have been experienced in most developed countries since the second half of the twentieth century, and this trend does not seem to slow down. Consequently, human longevity translates itself into a risk for annuity providers and pension funds, referred to as the longevity risk. As individuals live longer than expected, underestimating survival rates could lead to severe financial insolvencies. Therefore, the assessment of longevity risk needs to quantify the effect that mortality changes could have on the expected present value (EPV) of life annuities.

In the demographic literature, a considerable body of research has developed methods to assess changes in life expectancy when the age or time mortality functions change in shape or intensity. From a continuous-time perspective, refer to the papers of [1, 2], who provided a well-known formula that links changes over time in life expectancy at birth to a weighted total of rates of improvements in mortality or, more recently, to [3], who took a closer look at the life table entropy and provided additional insights for understanding how it relates to changes in mortality, or to [4], who studied the dynamic relationship between life expectancy and life span equality and numerically demonstrated that changes in both of them are weighted totals of rates of progress in reducing mortality.

This study aims to apply this approach to analyse the impact that changes in mortality could have on the life annuities EPV. As investigated by [5], the entropy measure can be used as an indicator to summarise the sensitivity of the cost of life annuities to changes in age-specific mortality, namely to determine the sensitivity of the life annuities EPV, for different interest rate scenarios. The study proves that, as for life expectancies, a decomposition formula can be obtained to express changes over time in the life annuities EPV, and a closed formula, which links the changes to the weighted average of improvement rates in mortality, is derived. The decomposition obtained is related to the entropy measure introduced in [5] and the duration index defined in [6].

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## Optimal Consumption and Investment Decisions with Disastrous Income Risk: Revisiting Rietz's Rare Disaster Risk Hypothesis

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We develop an analytically tractable dynamic model of optimal consumption and investment decisions with disastrous income risk in the context of Rietz's rare disaster risk hypothesis. We first empirically explore the relations among consumption changes, aggregate income, disaster shock severity, and fiscal measures in 55 countries during the Covid-19 period. We then by empirical motivation investigate an important role of insurance with a focus on the recovery of income in a disaster. We highlight how extent of disastrous income risk to which an agent is exposed and her income recovery post disaster jointly affect the agent's optimal decisions. Overall, availability of insurance can be particularly important for both the poor and the wealthy in the sense that they could even consume more, save less, and invest more post disaster as long as their future income is (partly) recovered.

## Why firms should buy insurance

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CT

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We show that frictions, such as corporate taxes and bankruptcy costs, can explain why firms buy insurance. Similar to corporate hedging, firm insurance reduces risk. Less risk allows for more use

of debt financing, increasing tax benefits of debt. More debt may increase bankruptcy costs. The increase in tax benefits dominates any increase in bankruptcy costs, resulting in a higher enterprise value. Moreover, optimal leverage increases and the cost of debt is reduced. These results are in line with standard corporate finance results.

## A machine learning approach based on survival analysis for IBNR frequencies in non-life reserving

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We introduce new approaches for forecasting IBNR (Incurred But Not Reported) frequencies by leveraging individual claims data, which includes accident date, reporting delay, and possibly additional features for every reported claim. A key element of our proposal involves computing development factors, which may be influenced by both the accident date and other features. These development factors serve as the basis for predictions. While we assume close to continuous observations of accident date and reporting delay, the development factors can be expressed at any level of granularity, such as months, quarters, or year and predictions across different granularity levels exhibit coherence. The calculation of development factors relies on the estimation of a hazard function in reverse development time, and we present three distinct methods for estimating this function: the Cox proportional hazard model, a feed-forward neural network, and xgboost (eXtreme gradient boosting). In all three cases, estimation is based on the same partial likelihood that accommodates left truncation and ties in the data. While the first case is a semi-parametric model that assumes in parts a log linear structure, the two machine learning approaches only assume that the baseline and the other factors are multiplicatively separable. Through an extensive simulation study and real-world data application, our approach demonstrates promising results. This paper comes with an accompanying R-package, `ReSurv`, [1].

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## Systematic longevity risk: The willingness to pay

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Increasing life expectancy has led to a global transition in pension systems towards more variable products in which risk is explicitly borne by the participants, necessitating a thorough understanding of longevity risk. This risk is explicitly transferred to policyholders contrasting with earlier implied hedges. Our goal is to quantify longevity risk through its impact on welfare, i.e., the willingness to

pay. Longevity risk can be categorized into idiosyncratic and systematic, with the latter, representing changes in life tables, being the focus of our study. The risk is determined as life expectancy changes over time beyond the already incorporated projected increase. Addressing the gap in quantifying systematic longevity risk, we introduce a multiple-horizon approach in which we calculate the realized “unexpected” deviations in best estimated survival rates due to the arrival of new observations in the mortality model. Our findings unveil that the willingness to pay to avoid the systematic longevity risk, i.e., the risk premium required to bear this risk, is substantial. We conduct extensive sensitivity analyses, exploring cross-country variations, different stochastic longevity models, and gender differentials, amongst others, contributing novel insights to the literature on the size of systematic longevity risk.

## Long-Term Mean-Variance Optimization Under Mean-Reverting Equity Returns

***Michael Preisel*** CT

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This paper [1] studies the mean-variance optimal portfolio choice of an investor pre-committed to a deterministic investment policy in continuous time in a market with mean-reversion in the risk-free rate and the equity risk-premium. In the tradition of Markowitz, optimal policies are restricted to a subclass of factor exposures in which losses cannot exceed initial capital and it is shown that the optimal policy is characterized by an Euler-Lagrange equation derived by the method of Calculus of Variations. It is a main result, that the Euler-Lagrange equation can be recast into a matrix differential equation by an integral transformation of the factor exposure and that the solution to the characteristic equation can be parametrized by the eigenvalues of the associated lambda-matrix, hence, the optimization problem is equivalent to a spectral problem. Finally, explicit solutions to the optimal policy are provided by application of suitable boundary conditions and it is demonstrated that - if in fact the equity risk-premium is slowly mean-reverting - then investors committing to long investment horizons realize better risk-return trade-offs than investors with shorter investment horizons.

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## Measuring proxy discrimination effects with sensitivity analysis

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Nowadays, increasing attention is given to potential discriminatory effects in actuarial pricing models [1]. Discrimination can manifest in two primary ways: directly, by explicitly incorporating discriminating covariates into the pricing model, or indirectly, by including covariates that are "permitted" but still contain information correlated with the discriminating ones. This latter scenario is called proxy discrimination, as (some) allowed covariates act as proxies for the discriminatory ones (as discussed by [2]). It becomes important, therefore, to quantify the proxy effect that

permitted covariates might have.

In this study, we propose a measure of this proxy effect based on sensitivity analysis. Specifically, we use the Shapley-Owen indices [4, 3]. By assessing the strength of the combined effects between the allowed and discriminating covariates on premiums, we derive a ranking of covariates that contribute most significantly to the proxy discrimination effect.

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[3] Plischke, E., Rabitti, G, Borgonovo, E. (2021). Computing Shapley Effects for Sensitivity Analysis. *SIAM/ASA Journal on Uncertainty Quantification* **9**, pp. 1411–1437.

[4] Rabitti, G, Borgonovo, E. (2019). A Shapley–Owen Index for Interaction Quantification. *SIAM/ASA Journal on Uncertainty Quantification* **7**, pp. 1060–1075.

## Risk profiles of Reverse Mortgage: empirical evidence from Italy

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Among the many challenges facing the world economies, population aging is no doubt one of the most important. According to World Bank Data ([13]), the old-age dependency ratio in the world increased from 11% in 2000 to 15% in 2022, and it is expected to increase to 32% by 2050. Both increasing life expectancy (+5.9%) and declining fertility rate (-0.32%) in the last twenty years contributed to World's rapid population aging. In addition, the inadequacy of pension systems to guarantee acceptable living standards and the large share in the European and North American population of so-called *home rich-cash poor* individuals are further motivations for the elderly to need to need more funding.

The Reverse Mortgage (RM) is a feasible solution for financial needs associated with longevity and it interacts directly with the elderly policyholder. Following Di Lorenzo et al. in ([4]), a RM contract allows (usually elderly) homeowners to receive a loan (a lump sum or a periodical cash flow) that will be repaid through the selling of her home following her death or her moving out of the home for any reason. Such a contract typically includes a no negative equity guarantee: if the loan balance exceeds the proceeds from the property's sale, the amount owed to the lender (e.g., the bank) is reduced to a (ante) fixed quantity. The lender is required to assess the level of risk in the RM contract by considering the lifetime of the holder, the value of the asset (the home), and the interest rate of the loan, which are all stochastic variables. Therefore, the RM contract is driven by three distinct risks: longevity risk, house price risk, and financial risk (see [10]). Recently in [5], the authors have used a closed formula ([3, 6]) to mathematically decompose the volatility

of the gain/loss for a lender in a RM contract through the above risks, and provided indexes for each component to assess their importance.

This paper intends to evaluate the effects of the risk quantification proposed in [5] on selected geographic regions of Italy in the period ranging from 2006 to 2023. A Quantitative Risk Analysis (QRA) was performed on data from databases regarding Demography, Health, Economics and Finance such as Agenzia delle Entrate, Istat, Cergas ([1, 2, 7]). The QRA contemplated the use of both parametric methods and non-parametric approaches for the purpose of prediction. The interest rates for loan-related valuations, the random survival function, and the interest rates related to the real estate value of homes were modeled using Geometric Brownian Motion ([12]), Lee Carter model ([9]), and Jump diffusion model ([8]). Additionally, a Monte Carlo simulation ([11]) was conducted twice: the first time to examine the robustness of various configurations generated via stochastic methodology described above, and the second time, to predict future variations of loan/gain. This study has several implications for scholars, practitioners, regulators and policymakers. The work contributes to: 1) bridge the gap between theoretical approaches during the early design phase of the financial instrument RM and empirical approaches during its actual applications; 2) give the lender a tool to assess and manage the RM contracts in Italy that are based on geographic areas: 3) improve the understanding and diffusion of RM contracts; 4) provide a list of possible scenarios for policymakers and regulators to take positive or negative actions.


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[5] Di Lorenzo, E., Rania, F., Sibillo, M., Trotta, A. (2024). Meeting the challenges of longevity: lifetime income from real estate. In press in: Corazza, M., Perna, C., Pizzi, C., Sibillo, M. (eds) Mathematical and Statistical Methods for Actuarial Sciences and Finance. MAF 2024. Springer, Cham.

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## Analyzing Extreme Weather Impact on Property Insurance Claim Severity: A Combined Pareto Neural Network Model

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In this study, we analyse the claim costs within the property insurance portfolio of an insurance company prompted by extreme weather-related events in Greece. We investigate their association with various property-insurance-related factors alongside geographical information and susceptibility to weather events like floods. In addition to conventional Pareto regression model, we employ advanced neural network (NN) modelling methodology to develop network-based model such as the combined actuarial neural network (CANN) approach of [1] in the context of weather-related claims size data. Specifically, we consider embedding the Pareto regression model within a neural network framework to model the claim amounts. We compare the predictive performance of the network-based model with underlying Pareto distributional assumption against the corresponding Pareto regression models to understand the impact of the NN models' capacity in capturing potential non-linear interactions within large and complex real-life datasets. Additionally, we also employ supplemental model improvement approaches such as early stopping and dropout for improved predictive performance. The practical utility of the proposed model lies in its ability to facilitate more informed rate setting within the property line of business. Moreover, their accuracy in forecasting, coupled with an in-depth comprehension of weather-related risks, could assist in developing disaster management preparedness programs for extreme weather events.

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## Deep Learning in Life Insurance

***Thorsten Rheinländer*** CT

Research Unit of Financial and Actuarial Mathematics, TU Wien, Austria

We discuss three applications of Deep Learning to insurance. Firstly, we implement a feed forward neural network for calculating solvency capital requirements. Secondly, we turn to deep mortality risk prediction where we use data from the Austrian pension insurance table for training. Finally, employing more complex stacked LSTM networks, we focus on mean-variance hedging for unit-linked life insurance. In all these topics, we discuss in detail the design of the network architecture.

## Likelihood-based estimation for multistate models subject to IBNR- and RBNS effects

***Kristian Buchardt*** [1,3], ***Christian Furrer*** [1], ***Oliver L. Sandqvist*** [1,2] CT

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[2] PFA Pension, Copenhagen, Denmark
[3] AP Pension, Copenhagen, Denmark

Life and health insurance policies generate longitudinal biometric data in the form of records on the occurrence and timing of certain events. Multi-state models provide a natural and parsimonious way to represent such data-generating mechanisms and are therefore commonly used for both pricing and reserving. However, complete observation of the event history is typically rendered impossible due to left-truncation and right-censoring, but also due to the presence of incurred-but-not-reported (IBNR) and reported-but-not-settled (RBNS) claims. While the latter mechanisms have received some attention in the non-life insurance literature under recurrent event models, see e.g. [1] and [2], the corresponding problem in life insurance is largely unexplored. In this talk, I will discuss how to accommodate these mechanisms in the estimation of a general multistate model using thinning-based methods to accommodate IBNR (reporting delays) and missing-data techniques to accommodate RBNS (incomplete event adjudication). We are motivated by the need for predictive models in pricing and reserving that capture trends in a timely fashion. The practical relevance is illustrated via a numerical study using simulations as well as via a data application based on a large Danish insurance portfolio that has been anonymized and slightly altered so as not to reveal any confidential information about the individual subjects or the insurance portfolio.

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## On Matrix-Valued Gamma Distributions in Multivariate Poisson Mixture Models

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CT

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In the collective model of actuarial science, it is popular to assume that the claim number has a Poisson distribution with a random intensity following a gamma distribution (which results in a negative binomial distribution allowing for overdispersion). The multivariate Poisson distribution enables an extension to a multi-business-line model with joint defaults. The vector of random Poisson intensities including their dependence structure can be modelled in terms of a matrix-valued gamma distribution, which is a generalization of the Wishart distribution. We discuss and derive properties of the involved distributions, including their degenerate variants. Particular emphasis is given to the probability-generating functions and the characterization of suitable biased probability distributions.

## Fair learning and testing for unfairness given protected features

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In the presence of a set of known protected features, we say that a predictor is fair if does not use, directly or indirectly, the effect of the protected features on the response. Within a causal framework, we show that this corresponds to a predictor that measures the average direct effect of the unprotected features on the response. This type of estimator is relevant in non-life insurance, where it is essential to remove discriminatory effects from prices.

In this talk, we accomplish two tasks: (a) We propose to derive a fair predictor via the following steps. One first estimates the direct effect of the protected features using double machine learning. Subsequently, this estimator is used to remove the effect of the protected feature on the response variable. Finally, any learning algorithm can be applied on the adjusted data. We conduct a small sample study that shows promising results. (b) In the second part, we introduce a measure of unfairness and develop a test that asymptotically has the right coverage. It takes any predictor as input and tests for unfairness on the original data.

## Optimal consumption, investment, and life insurance, including state dependence by risk-adjusted utility

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CT

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We investigate optimal consumption, investment, and life insurance problems by incorporating state dependence. State dependence typically expands state variables, leading to a more intricate problem. We work with an approach to state dependence that streamlines the problem rather than complicates the results and solutions. The idea is to quantify the financial worth of payments and compare these. Instead of evaluating the utility and, consequently, the moral value of the money, we take cues from derivative pricing techniques and compare payments by price or financial value. Within the financial state dependence, we further include the risk associated with life insurance and allow the agent to factor in the attitude to risk that the economic and insurance markets have established before making their decisions on life insurance, investments, and consumption. In the risk-adjusted framework, we present and interpret solutions to the optimal consumption, investment, and life insurance problem for general utility functions and CRRA (constant relative risk aversion) power utility functions.

## A Dirichlet Process Mixture regression model for the analysis of competing risk events

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CT

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[2] Mathematics and Computer Sciences, Eindhoven University of Technology, Netherlands

We present a regression modelling approach for the analysis of competing risk events. The joint distribution of the time to these events is flexibly characterized by a random effect which follows a discrete probability distribution drawn from a Dirichlet Process, explaining their variability. This entails an additional layer of flexibility of this joint model, whose inference is robust with respect to the misspecification of the distribution of the random effects. The model is analysed in a fully Bayesian setting, yielding a flexible Dirichlet Process Mixture model for the joint distribution of the time to events. An efficient MCMC sampler is developed for inference. The modelling approach is applied to the empirical analysis of the surrending risk in a US life insurance portfolio previously analysed by [2]. The approach yields an improved predictive performance of the surrending rates.

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## Second order Esscher pricing for models with jumps

***Tahir Choulli*** [2], ***Ella Elazkany*** [2], ***Michèle Vanmaele*** [1]

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The Esscher transform was introduced in the actuarial community by the seminal work of Esscher (1932). In their pioneering work Gerber and Shiu (1994) started a new era of applications by linking the Esscher transform and equivalent probability measures in option valuation. In particular in incomplete markets where infinitely many martingale measures exist equivalent to the physical measure describing the underlying price evolution, a popular choice for an equivalent martingale measure is based on the Esscher transform.

In this talk, we consider a stock process modelled by the exponential of a quasi-left-continuous semimartingale. Under a multidimensional framework, we introduce and define the *Esscher transform of the second order* in a continuous-time framework. As a first result we obtain that the second-order Esscher transform is a martingale measure and we provide a martingale condition. Different characterizations of the second-order Esscher martingale measure are given for multidimensional and one-dimensional semimartingales as well as for some general Lévy models. We show that for the same financial model we have *two characterizations* of the Esscher measure, the *exponential* Esscher measure and the *linear* Esscher measure, respectively. Next, we calculate the *Esscher pricing interval* of a European-style claim in terms of backward stochastic differential equations (BSDEs). Here, for a general jump-diffusion model we obtain that the problem of pricing a claim under the second-order Esscher can be reduced to studying a *constrained* BSDE problem. A problem that is well studied in the literature of portfolio optimization. When extending to pricing American-style claims and optimal stopping problems the Esscher pricing interval is given in terms of Reflected BSDEs.

In the special case of constant parameters the jump-diffusion model is reduced to a jump-diffusion with a constant jump model and an explicit formula for the European call option price is provided. For each choice of the free parameter, represented by the second-order, we can find a risk-neutral option price. We find that the lower price of the Esscher pricing interval is equal to the Black-Scholes price under the same volatility, interest rate and time to maturity. Some numerical applications under this model are conducted. We first compare the option price obtained for different values of the free parameter with other models, such as Black-Scholes, Merton's jump-diffusion, constant jump-diffusion model when jump risk is not priced, and the constant jump-diffusion when the jump risk is priced by the classical Esscher measure. We show that our Esscher pricing interval includes all prices given by the mentioned models except for some prices of the Merton's jump-diffusion model when the jump size volatility is high. We also conduct a sensitivity analysis of the interplay between model parameters and the Esscher free parameter and their effect on the option price. Finally, we experiment with real data. We fit different models to the log-return of the S&P 500 daily index using the maximum likelihood estimation method (MLE). It is evident that Merton's jump-diffusion model has a better fit and a more accurate option price compared to Black-Scholes, constant jump-diffusion (when jump-risk is not priced) and constant jump-diffusion under the classical Esscher measure. However, we show that under the second-order Esscher measure, there exists a value of the free parameter such that the price prediction error is minimized, and the accuracy outperforms Merton's jump-diffusion model.

## Estimation of Panel Models with Group Structures in Fixed Effects

***Enno Mammen*** [2], ***Ralf Wilke*** [1], ***Kristina Zapp*** [3]

CT

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[3] ZEW Mannheim, Germany

The recent econometrics literature studies models which are not identifiable because of unobserved individual effects. It has been proposed to use cluster analysis for the individual effects to get identification. In this paper, we consider a fixed effects panel model where the parameters on time-constant covariates are not identified. We present a new approach to clustering in this model to ensure identifiability. By using unsupervised nonparametric density-based clustering, cluster patterns including their location and number are adaptively determined by the model depending on the data. The approach works with large data structures. Our approach differs in two respects from the related literature. We allow for atoms, i.e. for units not belonging to a cluster and in our theoretical study we consider an asymptotic framework where the clusters are not consistently estimated in the limit. The performance of our method for large data sets is illustrated by simulations and an application to labour market data with 77,500 individuals and 620,000 person-year observations.

## Why You Should Not Trust Interpretations in Machine Learning: Adversarial Attacks on Partial Dependence Plots

***Xi Xin*** [1], ***Giles Hooker*** [2], ***Fei Huang*** [1]

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The adoption of artificial intelligence (AI) across industries has led to the widespread use of complex black-box models and interpretation tools. This paper proposes an adversarial framework to uncover the vulnerability of permutation-based interpretation methods for machine learning tasks, with a particular focus on partial dependence (PD) plots. This adversarial framework modifies the original black box model to manipulate model predictions for instances in the extrapolation domain, resulting in PD plots that can hide discriminatory behaviors while maintaining the prediction accuracy of the original model. This framework can produce multiple fooled PD plots via a single model. By using real-world datasets including an auto insurance claims dataset and COMPAS dataset, our results show that it is possible to intentionally hide the discriminatory behaviour of a predictor and make the black-box model appear neutral through interpretation tools like PD plots while retaining almost all the predictions of the original black-box model.

## Robust claim frequency modeling through phase-type mixture-of-experts regression

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CT

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[2] Department of Mathematical Sciences, University of Copenhagen, Denmark

In this talk, we address the problem of modeling loss frequency using regression when the counts have a non-standard distribution. We propose a novel approach based on mixture-of-experts specifications on discrete-phase type distributions. Compared to continuous phase-type counterparts, this approach offers fast estimation via expectation-maximization algorithms, making it more feasible for use in real-life scenarios. Our model is both robust and interpretable in terms of risk classes and can be naturally extended to the multivariate case through two different constructions. Using simulated and real-world data, we showcase how our approach provides an effective solution for modeling loss frequency in non-standard situations.

[1] Bladt, M., and Yslas, J. (2023). Robust claim frequency modeling through phase-type mixture-of-experts regression. *Insurance: Mathematics and Economics* **111**, pp. 1–22.

## Isotonic distributional regression and CRPS decompositions

***Johanna Ziegel***

PS

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Isotonic distributional regression (IDR) is a nonparametric distributional regression approach under a monotonicity constraint. It has found application as a generic method for uncertainty quantification, in statistical postprocessing of weather forecasts, and in distributional single index models. IDR has favorable calibration and optimality properties in finite samples. Furthermore, it has an interesting population counterpart called isotonic conditional laws that generalize conditional distributions with respect to $\sigma$-algebras to conditional distributions with respect to $\sigma$-lattices. In this talk, an overview of the theory is presented. Furthermore, it is shown how IDR can be used to decompose the mean CRPS for assessing the predictive performance of models with regard to their calibration and discrimination ability.

## Optimal dividend payout under path-dependent constraint

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CT

This paper is concerned with a long standing optimal dividend payout problem in insurance subject to the so-called ratcheting constraint, that is, the dividend payout rate shall be non-decreasing over time. The surplus process is modeled by a drifted Brownian motion process and the aim is to find the optimal dividend ratcheting strategy to maximize the expectation of the total discounted dividend payouts until the ruin time. We propose a novel partial differential equation method to solve the problem and derive an optimal dividend ratcheting strategy, and thus solve the open problem completely. Economically, we find that if the surplus volatility is above an explicit threshold, then one should pay dividends at the maximum rate, regardless the surplus level. Otherwise, by contrast, the optimal dividend ratcheting strategy relays on the surplus level and one should only ratchet up the dividend payout rate when the surplus level touches the dividend ratcheting free boundary. The full paper is available at [1].

[1] Guan C, Xu Z. Q. (2023). Optimal ratcheting of dividend payout under Brownian motion surplus, *arXiv:2308.15048*.

## Heterogeneous extremes in the presence of random covariates and censoring

***Martin Bladt***, ***Christoffer Øhlenschlæger***

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CT

The task of analyzing extreme events with censoring effects is considered under a framework allowing for random covariate information. A wide class of estimators that can be cast as product–limit integrals is considered, for when the conditional distributions belong to the Fréchet max-domain of attraction. The main mathematical contribution is establishing uniform conditions on the families of the regularly varying tails for which the asymptotic behaviour of the resulting estimators is tractable. In particular, a decomposition of the integral estimators in terms of exchangeable sums is provided, which leads to a law of large numbers and several central limit theorems. Subsequently, the finite-sample behaviour of the estimators is explored through a simulation study, and through the analysis of two real-life datasets. In particular, the inclusion of covariates makes the model significantly versatile and, as a consequence, practically relevant.